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Grey Entropy Quantum-behaved Chaotic Particle Swarm Optimization Based on High-dimension Multi-objective Optimization Design of Mixed Discrete Variables

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ABSTRACT

A grey entropy quantum-behaved chaotic particle swarm optimization has been proposed based on the effective solution of high-dimension multi-objective optimization design of mixed discrete variables. By combining grey entropy correlation analytical method and theory of information entropy, this work proposed the distribution strategy of the grey entropy relational adaptive value. With the adaptive value, calculation results of grey entropy correlation coefficient and entropy weight, the high-dimensional multi-objective optimization problem can be transformed into single objective optimization. Thereby, it can lead the evolution of heuristic algorithm using grey entropy relational value. Quantum-behaved particle swarm optimization has the shortcomings of slow convergence as well as easily getting into local minimum. Dynamic penalty function was constructed using the premature judgment based on group fitness variance premature judgment mechanism. Then adaptive value can be calculated by CPSO, thus improving the search efficiency. In this way, a quantum-behaved chaotic quantum particle swarm optimization (MOPTDCQPSO1.0) with high-dimension multi-objective optimization was developed based on mixed discrete variables. Optimization examples show that this algorithm has no special requirements for the characteristics of optimization design, with good universality, reliable operation and strong convergence. Furthermore, it can also easily and effectively solve the problems of high-dimension multi-objective optimization for mixed discrete variables and robust design.

KEYWORDS: Mixed discrete variables; Grey entropy correlation; High-dimensional and multi-objective; Dynamic penalty function; Quantum-behaved particle swarm optimization; Chaos search

INTRODUCTION

In engineering optimization design, there are always the problems of non-continuous variables, such as integer variables (number of teeth) and discrete variables (modulus of gear). For the optimization design of mixed discrete variables, mixture of integer variable, discrete variable and continuous variable is of universal engineering significance. However, as the difficulty in mathematical programming and operational research, some shortcomings and difficulties still exist in discrete variable optimization. In practical applications, because the objective functions in multi-objective optimization problem are always not comparable or even conflicting, it is difficult to use classical optimization methods to solve them [2]. Evolutionary computation is a technique based on population operations, which can improve solving ability by using the similarity of different solutions. Therefore, it is suitable for solving the optimization of multiple objects. PSO , another evolutionary computation, has been widely applied in many optimization problems for its simplicity and operability. Besides, such algorithm is more efficient than genetic algorithm in certain conditions. But there is also premature existing in PSO algorithm, so it is hard to obtain the optimal solution for complex issues. Recently, research mainly aimed at extending PSO to solve the problems of multiple objectives. However, these methods can only be used for the optimization of two objective functions, with difficulty to solve the problems of three or more objectives with large computation [4]. Grey system theory is a new theory proposed by Chinese scholar Professor Deng Julong. And it has been widely used in many fields after 20 years of development [5]. Luo Youxin et al. presented the grey clustering, grey entropy correlation and grey decision method of multi-objective optimization to evaluate multiple targets [6]. Chen Yibao et al. focused on the application of Deng's correlation for the multi-objective optimization design in pumping, and optimum results were obtained [7]. However, the multi-objective optimization design has a long way to go [8-10]. In this work, the distribution strategy of grey entropy relational adaptive value was proposed by combining grey entropy correlation analytical method and theory of information entropy. With the adaptive value, calculation result of grey entropy correlation coefficient and entropy weight, the high-dimensional and multi-objective optimization problem can be transformed into single objective optimization. Thereby, it can lead the evolution of heuristic algorithm using grey entropy relational value. Quantum-behaved particle swarm optimization has the shortcomings of slow convergence and easily getting into local minimum. Using the premature judgment based on group fitness variance premature judgment mechanism, dynamic penalty function was constructed. Then adaptive value can be calculated by CPSO, thus improving the search efficiency. In this way, a quantum-behaved chaotic quantum particle swarm optimization (MOPTDCQPSO1.0) with high-dimension multi-objective optimization was developed based on mixed discrete variables. Optimization examples show that this algorithm has no special requirements for the characteristics of optimization design, with good universality, reliable operation and strong convergence. Furthermore, it can also easily and effectively solve the problems of high-dimension multi-objective optimization for mixed discrete variables and robust design.

OPTIMIZATION DESIGN METHOD BASED ON GREY ENTROPY CORRELATION

High-dimension multi-objective optimization design model

Following multi-objective nonlinear problems should be considered.

$$\min F(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_k(\mathbf{x})\}^T, (r = 1,2,\cdots,k)$$
$$S.t. \quad g_i(\mathbf{x}) \le 0, i = 1,2,\cdots m$$
$$h_q(\mathbf{x}) = 0, q = 1,2,\cdots, p \tag{1}$$

where $\mathbf{x} = [x_1, x_2, \cdots, x_n]^T$ refers to design variables; n the number of variables; $f_r(\mathbf{x})$ $(r = 1,2,\cdots,k)$ the r-th object function; $g(\mathbf{x})$ the inequality constraint; $h(\mathbf{x})$ the equality constraint.

Solution to grey entropy correlation of multi-objective optimization design

Grey system theory aims at studying the uncertainty problems with small sample or poor information. In the grey system, correlation indicates the similar degree of dynamic development among things or factors. The larger the correlation, the more similar of things is, vice versa. Traditional method of mathematical statistics usually requires large sample and huge computation, or even lead to quantitative results are not consistent with qualitative analysis. Compared with traditional method, grey system theory is more simple and convenient.

It is hypothesized that baseline vector sequence is $\mathrm{F}_{00} = (F_{001}, F_{002}, \cdots, F_{00M})$, and M stands for the numbers of objective function. The objective vector sequence is $\mathrm{F}_{01} = (F_{01}, F_{02}, \cdots, F_{0M})$, then F_{00} and F_{01} are processed as follows:

$$\varepsilon_k = \frac{a + \rho b}{\left|\mathrm{F}(k) - \mathrm{F}_0(k)\right| + \rho b} \tag{2}$$

where $a = \min(\mathrm{F} - \mathrm{F}_0)$, $b = \max(\mathrm{F} - \mathrm{F}_0)$, and the average of correlation is $\varepsilon = \sum_{k=1}^{N} \varepsilon_K$.

Assuming $P_k = \dfrac{F(k)}{\sum_{k=1}^{n} F(k)}$, information entropy of each objective is:

$$e_k = -\frac{1}{n} \sum_{k=1}^{n} P_k \ln P_k . \tag{3}$$

The entropy weight for each objective is:

$$w_k = \frac{1 - e_k}{\sum_{k=1}^{n} (1 - e_k)} . \tag{4}$$

Then the grey entropy correlation can be obtained as follows.

$$r_k = \sum_{k=1}^{n} w_k \varepsilon_k . \tag{5}$$

Compared with Deng's correlation, grey entropy correlation has many characteristics. One of the important characteristics is that grey entropy correlation *r* will change correspondingly with the change of either F or F_0. This is of significance for iteration in optimization design. Besides, as a result of initial treatment, the magnitude difference of F_{00} and F_{01} of each objective has little influence on optimization. As baseline sequence F_{00} is generally unknown in advance, the minimum value of corresponding objective function is taken as the baseline sequence dynamically in iteration. Computing process of grey entropy correlation shows that this method aims at utilizing the information among each objective of Pareto solution from the proximity of each objective among Pareto and ideal solutions. In this way, this method can take into account the relationship between various objectives within Pareto solution, as well as the actual distance between Pareto solution and ideal solution. Therefore, better solution of grey entropy correlation can be obtained when dealing with multi-objective optimization problems. Grey entropy correlation is a global method to analyze whole proximity during the dynamic development of grey entropy. In this work, grey entropy correlation was taken as the adaptive value of Pareto solution computed in multi-objective optimization. Then the high-dimensional and multi-objective optimization problem was transformed into single objective optimization. Thus, it can lead the evolution of heuristic algorithm using grey entropy correlation.

CHAOTIC QUANTUM PARTICLE SWARM OPTIMIZATION ALGORITHM BASED ON MIXED DISCRETE VARIABLES

Quantum particle swarm algorithm

Basic particle swarm algorithm

Particle Swarm Optimization (PSO), a global optimization algorithm, was proposed in recent years. Kennedy and Eberhart [11, 12], the designers of POS, were inspired by the foraging behavior of animals. They discovered that while searching for optimal target, each individual will adjust next search by the best one of population or best position itself has ever reached. Particle swarm algorithm was designed based on this founding. And the iterative Equations of basic PSO algorithm are shown as follows.

$$\mathbf{v}_{id}(t+1) = w\mathbf{v}_{id}(t) + c_1\phi_1(Pbest_{id}(t) - \mathbf{x}_{id}(i)) + c_2\phi_2(Gbest_{id}(t) - \mathbf{x}_{id}(t)) \quad (6)$$

$$\mathbf{x}_{id}(t+1) = \mathbf{x}_{id}(t) + \mathbf{v}_{id}(t+1) \quad (7)$$

Among them, v and x are the speed and position of each particle, respectively; i and d the serial number of particle and component number, respectively; t the steps of iterative calculation; w, c_1 and c_2 the system control parameters. The value of c_1 and c_2 is generally between 1 and 2. ϕ_1 and ϕ_2 are the random numbers uniformly distributed on the interval $[0,\ 1]$. $Pbest_{id}$ is the optimal location ever reached by particles, and $Gbest_{id}$ is the optimal location of the particle swarm. It can be seen that each individual can do it better by making full use of intelligence of the swarm and itself. The idea of PSO is to obtain satisfactory solution.

To reduce the likelihood of leaving the search space, the flight speed of particles is usually limited to less than the maximum $V_{\max}$. If the $V_{\max}$ is too large, particles will fly over the optimal solution. While $V_{\max}$ is too small, particles cannot effectively explore the area outside local optimum solution. Furthermore, particles may fall into local extreme, so it is difficult to fly to further place to search for optimal solution. Usually, the value was taken as $V_{k,\ d} \in [-V_{\max},\ V_{\max}]$, and $V_{\max}$ was set by users.

In basic PSO algorithm, the convergence of particles is achieved in the form of track. And for the limited velocity of particles, the search space of particles is usually limited, thus hardly covering the whole of feasible space. Therefore, standard PSO algorithm cannot incontrovertibly guarantee a convergence to global optimal solution, which is the biggest drawback of standard PSO algorithm.

Quantum-behaved particle swarm optimization

In 2004, after studying the research results of Clerc on converge behavior of particles, Sun et al. proposed a new PSO algorithm model from the angle of quantum mechanics [13, 14]. This model is based on DELTA potential well, holding the view that particles exhibit quantum behavior. On this base, quantum-behaved particle swarm optimization was proposed. In quantum space, completely different with aggregation in nature, particles can search the whole of feasible space. Thus, global searching performance of QPSO is far superior to standard PSO algorithm.

In QPSO algorithm, the population size was supposed as M. During generation, particles will be added or reduced within certain probability to update the position of each particle to generate new particle swarm. The position was shown as follows.

$$p = a * Pbest(i) + (1-a) * Gbest \tag{8}$$

$$mbest = \frac{1}{M}\sum_{i=1}^{M} Pbest(i) \tag{9}$$

$$b = 1 - \frac{generation}{\max generation} * 0.5 \tag{10}$$

$$position = \begin{cases} p - b * \left| mbest - position \right| * \ln(1/u) & (u \ge 0.5) \\ p + b * \left| mbest - position \right| * \ln(1/u) & (u < 0.5) \end{cases} \tag{11}$$

where a, u were random numbers between 0 to 1; $mbest$ the average of particle swarm $pbest$; b, the contraction coefficient, will decrease during convergence; $generation$ the evolutionary generations currently; $\max generation$ the maximum evolutionary generations set by users.

Chaos quantum-behaved particle swarm optimization

Judgment mechanism of premature based on variance of swarm's fitness value

During operation of QPSO, once an optimal position was found by particle, the others will quickly draw close to this position. If the optimal position is local minima, algorithm will fall into local optimum, thus resulting into premature convergence. Experiments show that whether QPSO appears premature or global convergence, particles swarm will all aggregate. The particles should gather at certain particular location, or several particular points. It mainly depends on the characteristics of problem itself and selection of fitness function. In order to quantitatively describe the state of particle swarm, the fitness variance of swarm is as follows.

$$\sigma^2 = \frac{1}{M}\sum_{i=1}^{M}\left(\frac{f_i - \bar{f}}{\max(1, \max\left|f_i - \bar{f}\right|}\right) \qquad (i = 1, 2, \cdots, M) \tag{12}$$

where the fitness variance of swarm σ^2 reflects the converging degree of all particles. Only small σ^2 can lead to the convergence of particles, otherwise they will continue to research randomly. Literature [7] proved that if particle swarm falls into premature or global convergence, the particles will gather in one or several specific positions of search space. Besides, the fitness variance of swarm is equal to zero.

Dynamic penalty function

As methods for solving unconstrained optimization problems are various, one idea is to transform constraint problems into unconstrained ones. Among them, penalty function is the most extensive method. The specific approach is to construct some penalty function based on the characteristics of constraint. Then, adding it to objective function, the constrained optimization problems can be solved as unconstrained ones. Such penalty strategy will offer a large objective function value to iteration point which attempt to violate constraint. While for minimum point, it is a kind of penalty. Because this method will force minimum points without constrained problems to either infinitely approach feasible region or continue moving within feasible region. This process will continue until the iterative sequences have converged to minimum point of original constraint problem.

After transformation, the unconstrained optimization problem is shown as follows.

$$G(x) = F(x) + H(x)h(t), \tag{13}$$

where $F(x)$ is the fitness function; $h(t)$ the force of punishment; t the evolution generations; $H(x)$ the penalty factor. It can be defined as Equation (14).

$$H(x) = \sum_{i=1}^{m} \mu(\phi_i(x))\phi_i(x)^{\delta(\phi_i(x))}, \tag{14}$$

where $\varphi_i(x)=\max\{0,g_i(x)\}$; $g_i(x)$ represents the constraint function; $h(\bullet)$, $\mu(\bullet)$ and $\delta(\bullet)$ depend on specific situations.

Chaos quantum-behaved particle swarm optimization

Chaos is a kind of irregular movement, which means random behavior appears without any additional random factors in deterministic non-linear systems, namely inner couplers randomness. The greatest characteristic of chaotic systems is that the evolvement of system is very sensitive to initial value. Therefore, the future behavior of system is unforeknown in long-term. With these characteristics, chaotic system can be used in optimization calculation. And due to its regularity, new solutions can be generated by determined iteration equation, thus making programming very easy. Randomness can prevent the search from falling into local optimum. Most importantly, if the ergodicity property of chaotic system can be controlled properly, the final solution can approach the optimal solution at any precision. Random number generated by common methods cannot complete traversal search of continuously variable space, because it is impossible to reach any precision without repetition. But chaos variables can make it within the precision range of computer. Chaos Equation used in this work is Hénon image, a typical hyper-chaotic system, and the expression is as follows.

$$\begin{Bmatrix} z_{1,k+1} \\ z_{i,k+1} \end{Bmatrix} = \begin{Bmatrix} a_1 - z_{n-1,k}^2 - b_1 z_{n,k} \\ z_{i-1,k} \end{Bmatrix} \tag{15}$$

where $i = 2,3,\cdots,n_1$ represents the dimensions of system; k the discrete time; a_1 and b_1 the adjustable parameters. When $i = 2$, the image above is the famous Hénon image. When $a_1 = 1.76, b_1 = 0.1$, the calculation have been carried out in literature. Results show that with the increase of n_1, there is a simple relationship between the number of Lipschitz exponent n (greater than zero) and the dimension of system n_1, namely $n_1 = n+1$.

According to the characteristic of chaotic motion, chaos quantum-behaved particle swarm optimization can better escape premature convergence as well as improve the ability of local detailed development. For the sensitivity of chaos to initial value, n initial values $z0_i(i = 1,2,\cdots,n)$ with slight difference were substituted into Equation (8) for several iterations, respectively. Then, n chaotic variables are obtained. Finally, these variables will be imaged to the value space of variables with optimization design, thus obtaining $\mathbf{x} = [x_1, \cdots, x_i, \cdots, x_n]^{\mathrm{T}}$.

$$x(i) = LB(i) + (UB(i) - LB(i))z(i) \tag{16}$$

For the corresponding objective function value of $\mathbf{x}$, if $f \leq Pbest$, then $Pbest = f$.

The flow of chaos quantum-behaved particle swarm optimization is as follows:

1) Initializing population;

2) Contraction coefficient b linearly reduces from 1.0 to 0.5 with the increase of iteration times;

3) The average of population's best position $mbest$ is calculated based on Equation (4);

4) Each particle's fitness value is compared with its best position $Pbest_i$. If the former is better, the fitness value will be taken as the best position of individual currently;

5) Each particle's fitness value is compared with the global best position $Gbest$. If the former is better, then it will be taken as the global best position;

6) The new position of particles is refreshed based on the evolutionary Equation (6) of QPSO algorithm;

7) The variance σ^2 of population's fitness is calculated according to Equation (7). Then whether the population has fall into premature can be judged by $\sigma^2 \leq C$. The value of C depends on specific situations. Then, N times of chaotic search will be carried out according to Equation (11) to obtain the optimization solution vector $\mathbf{x}^*$ and corresponding adaptive value $Pbest$. If $Pbest < Gbest$ (the adaptive value of global optimization), then $Gbest = Pbest$;

8) If the termination condition is not satisfied, the algorithm will return to step (2). Otherwise, the algorithm is over.

Engineering treatment method on designing variables

Discretization of discrete design variables

In grey entropy quantum chaotic particle swarm optimization algorithm, as the updated new individuals are continuous variables, discretization for the population is necessary after each operation. Discretization for integral design variables is similar to that for non-equidistant discrete variables. The difference lies in the value space, which is non-negative integer between fixed upper-lower limit.

Engineering treatment on continuous design variables

In engineering optimization design, although some design variables have continuous forms, their values are still restricted by machinery manufacturing precision and design specifications. In optimization design, the variables were calculated as the real number of floating point or double precision of programming language. After optimization, the results were processed according to the required decimal places of engineering. If processing like this, the ultimately design may not be the optimal solution, or even cannot satisfying the constraints.

Designed program

Through above algorithm, the high-dimension multi-objective optimization design chaotic quantum particle swarm optimization (MOPTDCQPSO1.0) based on the solution of grey entropy correlation was developed by constructing dynamic penalty function, using grey entropy correlation and processing method for mixed variables.

APPLICATION EXAMPLES

Example 1: To verify the correctness of this method, multi-objective optimization was carried out for valve spring presented in literature [18]. Spring is required to with lightest weight, smallest free height and largest natural frequency. The optimization model is as follows.

$$\mathbf{x} = [x_1, x_2, x_3]^{\mathrm{T}}$$
$$f_1(\mathbf{x}) = x_1^2 x_2 (x_3 + 1.8) \to \min$$
$$f_2(\mathbf{x}) = x_1 (x_3 + 1.3) \to \min$$
$$f_3(\mathbf{x}) = 3.56 \times 10^5 \frac{x_1}{x_2^2 x_3} \to \max$$

S.t.

$$g_1(\mathbf{x}) = 6.5 - |x_2 / x_1 - 9.5| \geq 0$$
$$g_2(\mathbf{x}) = 0.01 - \left|10^4 x_1^4 x_2^{-3} x_3^{-1} / 47 - 1\right| \geq 0$$
$$g_3(\mathbf{x}) = 405 - 2771 x_1^{-2.86} x_2^{0.86} \geq 0$$
$$g_4(\mathbf{x}) = 3.74286 - \frac{(x_3 + 1.3)x_1 + 18.25}{x_2} \geq 0$$
$$g_5(\mathbf{x}) = 3.56 \times 10^5 x_1 x_2^{-2} x_3^{-1} - 250 \geq 0$$
$$g_6(\mathbf{x}) = x_3 - 3 \geq 0$$
$$g_7(\mathbf{x}) = x_2 - 30 \geq 0$$
$$g_8(\mathbf{x}) = 60 - x_2 \geq 0$$
$$g_9(\mathbf{x}) = x_1 - 2.5 \geq 0$$
$$g_{10}(\mathbf{x}) = 9.5 - x_1 \geq 0$$

As shown above, $f_1(\mathbf{x})$ refers to the objective function of spring's weight; $f_2(\mathbf{x})$ the objective function of spring's height; $f_3(\mathbf{x})$ the objective function of spring's natural frequency; x_1, x_2, x_3 the diameter, in-diameter and effective laps, respectively. These discrete variables have all been processed according to national standard. Table 1 shows the results after three times of optimization by MOPTDEMPCOA. It can be seen that the solutions in this work are better than literature [18].

Table 1: Parameters of optimization design and the comparison

Design parameters	x_1	x_2	x_3	$f_1(\mathbf{x})$ (min)	$f_2(\mathbf{x})$ (min)	$f_3(\mathbf{x})$ (max)
Solution in literature [18]	5.5	31	6.5	7783	42.9	313.5
Solution 1 in this work	5.5	34	5	6993.8	34.65	338.75
Solution 2 in this work	6	45	3	7776	25.8	351.60

Example 2: In optimization design for gear reducer, the optimization model for a single-stage reducer in literature [1] is as follows.

$$\min \quad f(\mathbf{x}) = 0.78539815(4.75x_1x_2^2x_3^2 + 85x_1x_2x_3^2 - 85x_1x_3^2 + 0.92x_1x_6^2 - x_1x_5^2 + 0.8x_1x_2x_3x_6 - 1.6x_1x_3x_6 + x_4x_5^2 + x_4x_6^2 + 32x_6^2 + 28x_5^2)$$

$$s.t. \quad g_1(\mathbf{x}) = 17 - x_2 \le 0$$

$$g_2(\mathbf{x}) = x_2x_3 - 30 \le 0$$

$$g_3(\mathbf{x}) = 0.2 - x_3 \le 0$$

$$g_4(\mathbf{x}) = 16 - x_1 / x_3 \le 0$$

$$g_5(\mathbf{x}) = x_1 / x_3 - 35 \le 0$$

$$g_6(\mathbf{x}) = 10 - x_5 \le 0$$

$$g_7(\mathbf{x}) = x_5 - 15 \le 0$$

$$g_8(\mathbf{x}) = 13 - x_6 \le 0$$

$$g_9(\mathbf{x}) = x6 - 20 \le 0$$

$$g_{10}(\mathbf{x}) = x_1 + 0.5x_6 - x_4 + 4 \le 0$$

$$g_{11}(\mathbf{x}) = \frac{43854}{x_2x_3\sqrt{x_1}} - 855 \le 0$$

$$g_{12}(\mathbf{x}) = \frac{7098}{x_1x_2x_3^2(0.169 + 0.006666x_2 - 0.0000854x_2^2)} - 261 \le 0$$

$$g_{13}(\mathbf{x}) = \frac{7098}{x_1x_2x_3^2(0.2824 + 0.00177x_2 - 0.0000394x_2^2)} - 213 \le 0$$

$$g_{14}(\mathbf{x}) = \frac{0.01233x_4^3}{x_1x_3x_5^4} - 0.003x_4 - 261 \le 0$$

$$g_{15}(\mathbf{x}) = 29050\,\frac{29050x_4}{x_2x_3x_5^3}\sqrt{1 + \frac{0.29709x_2^2x_3^2}{x_4^2}} - 55 \le 0$$

$$g_{16}(\mathbf{x}) = \frac{29050x_4}{x_2x_3x_6^3}\sqrt{1 + \frac{7.42727x_2^2x_3^2}{x_4^2}} - 55 \le 0$$

As shown in above model, objective function $f(x)$ (cm^3) refers to the volume of reducer; x_1 (cm) the width of gear; x_2 (cm) the number of teeth of little gear; x_5 and x_6 (cm) the diameters of two gears, respectively. These four variables are all integer variables. x_3 represents the modulus of gear. The first series of standard modulus are taken as discrete variables. As a continuous variable corrected to two decimal places, x_4 indicates the width of gear case. The rate of change of objective function for each variable is the minimum. Considering this, the optimization problem of gear can be transferred into multi-objective optimization problem to improve the robustness of results. That can be seen as follows:

$$\min \mathbf{F}(\mathbf{x}) = [f(\mathbf{x}) \quad \left|\frac{\partial f(\mathbf{x})}{\partial x_1}\right|, \quad \left|\frac{\partial f(\mathbf{x})}{\partial x_2}\right|, \quad \left|\frac{\partial f(\mathbf{x})}{\partial x_3}\right|, \quad \left|\frac{\partial f(\mathbf{x})}{\partial x_4}\right|, \quad \left|\frac{\partial f(\mathbf{x})}{\partial x_5}\right|, \quad \left|\frac{\partial f(\mathbf{x})}{\partial x_6}\right|]^{\mathrm{T}}$$

Its constraint function is similar to that of single objective optimization function $f(\mathbf{x})$. Using the method in this work, the solving results are shown in Table 2 and Table 3. It can be seen that the optimum solution is better than that solved by existing methods.

Table 2: Design parameters of the optimization of gear reducer

Design parameters	x_1/cm	x_2	x_3	x_4/cm	x_5/ cm	x_6/cm
Solution in literature [1]	13	24	0.60	23.75	10	13
Solution 1 in this work	4	130	0.2	21.62	10	13
Solution 2 in this work	4	130	0.2	17	10	13

Table 3: Value of objective function of the optimization of gear reducer

Objective functions	$f(x)$ /cm^3	$\left\lvert\frac{\partial f}{\partial x_1}\right\rvert$	$\left\lvert\frac{\partial f}{\partial x_2}\right\rvert$	$\left\lvert\frac{\partial f}{\partial x_3}\right\rvert$	$\left\lvert\frac{\partial f}{\partial x_4}\right\rvert$	$\left\lvert\frac{\partial f}{\partial x_5}\right\rvert$	$\left\lvert\frac{\partial f}{\partial x_6}\right\rvert$
Solution in literature [1]	30675	1478	1214	59811	211	609	1490
Solution 1 in this work	23490	3119	172	118837	211	716	1234
Solution 2 in this work	22514	3119	172	118837	211	644	1140

CONCLUSIONS

(1) An entropy quantum-behaved chaotic particle swarm optimization has been proposed based on the effective solution of high-dimension multi-objective optimization design with mixed discrete variables. By combining grey entropy correlation analytical method and theory of information entropy, this method proposed the distribution strategy of the grey entropy relational adaptive value. With the adaptive value, calculation result of grey entropy correlation coefficient and entropy weight, the high-dimensional and multi-objective optimization problem can be transformed into single objective optimization. Thereby, it can lead the evolution of heuristic algorithm using grey entropy relational value. A new method based on chaos search was proposed by using the premature judgment based on group fitness variance premature judgment mechanism and constructing dynamic penalty function, thus improving the search efficiency. Then, this work constructs the chaotic quantum-behaved particle swarm optimization algorithm with high-dimension multi-objective optimization design of mixed discrete variables.

(2) In this work, the high-dimension multi-objective optimization design chaotic quantum particle swarm optimization (MOPTDCQPSO1.0) was programed. This software can rationally process the value problem of mixed discrete variables in optimization design.

(3) This algorithm has no special requirements for the characteristics of optimization design, with good universality, reliable operation and strong convergence. Furthermore, it can also easily and effectively solve the problems of optimization and robust design of mixed discrete variables.

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